מכון טכנולוגי חולון
Holon Institute of Technology

COST ACTION TD 1409
Mathematics for Industry Network

Industrial Day at HIT

On June 29, 2017, we held an industrial day at HIT Holon Institute of Technology. The main goal of this event was to increase activity of industrial mathematics in Israel by bringing together academic and industrial researchers/practitioners providing the opportunity for networking, exchanging ideas, problem solving, and/or identification of areas for collaboration. In particular we were gathered to model and solve a real industrial problem introduced to the participants by Dr. Liat Peled from Rafael Advanced Defense Systems Ltd.

The participants were gathered from different mathematical and engineering background and different European countries. In total, there were 11 leading researchers:

- Prof. Vassilis Kostoglou, Alexander Technological Educational Institute of Thessaloniki, Greece
- Prof. Adir Pridor, HIT
- Prof. Reinier Díaz Millán, Federal Institute of Goiás, Brazil
- Prof. Andrij Trokhymchuk, Institute for condensed matter physics, Ukraine
- Dr. Aviv Gibali, ORT Braude College
- Dr. Liat Peled, Rafael Advanced Defense Systems Ltd
- Dr. Yirmeyahu Kaminski, HIT
- Dr. Tomer Shushi, Ben-Gurion University, Beer-Sheva, Israel
- Mr. Oleg Kelis, University of Haifa and Ort Braude College
- Prof. Anatoly Golberg, HIT
- Prof. Ezra Zeev, HIT

The problem which was introduced by Dr. Liat Peled can be summarized as follows:

גולומב 52, ת.ד 305, חולון 5810201
טלפקס: 03-5026560
52 Golomb St., Holon 5810201 Israel
www.hit.ac.il Tel/Fax: 972-3-502-6560
הפקולטה למדעים
המחלקה למתמטיקה שימושית
Faculty of Sciences
Department of Applied Mathematics

When can a non-linear system be defined in an extended linear way such that the resulting system is observable?

More precisely consider a non-linear system $\begin{cases} \dot{x} = f(x) \\ y = h(x) \end{cases}$*, where* $x \in R^n$ *is a state vector and* $y \in R^m$ *a measurement vector. Can we find matrix valued functions* F, H *such that the system can be written in the following form* $\begin{cases} \dot{x} = F(x)x \\ y = H(x)x \end{cases}$ *and* $\operatorname{rank}([H, HF, \cdots HF^{n-1}]) = n$*?*

During this day we held very fruitful discussions in which we managed to get some insights which contribute to a better understanding of the problem as well as had preliminary numerical illustrations with Maple.

We shall present here briefly the approach that has been designed to solve the problem. Basically we have two systems for a given x: $f(x) = F(x)x$ and $h(x) = H(x)x$. These systems are linear in the entries of the matrices $F(x)$ and $H(x)$. Unless there is no solution at all, this define two affine varieties: $F(x) = F_0 + \sum \alpha_j F_j$ and $H(x) = H_0 + \sum \beta_j H_j$. Note that the directions F_j and H_j only depend on the order of the system, i.e. the dimensions n of the state space and m of the output space.

Then we form the observability matrix:

$$C = \begin{bmatrix} H \\ HF \\ \vdots \\ HF^{n-1} \end{bmatrix},$$

Where n is the state dimension. For the system to be observable, the matrix C must have maximal rank, which is equivalent to require that at least one maximal minor doesn't vanish. This defines an open condition on $(\alpha_j)_j$ and $(\beta_j)_j$. Values of α_j and β_j that satisfy this open condition define extended linear forms that makes the system observable.

Since this is an open condition, some optimization can be performed to find the optimal values of α_j and β_j. Some of the participants still work on this refinement.