Isogeometric modeling of Lorentz detuning in linear particle accelerator cavities

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PhD Modelling Week
September 10, 2016

Outline

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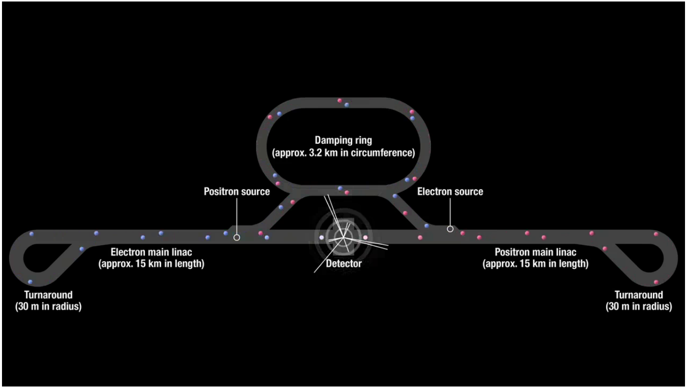
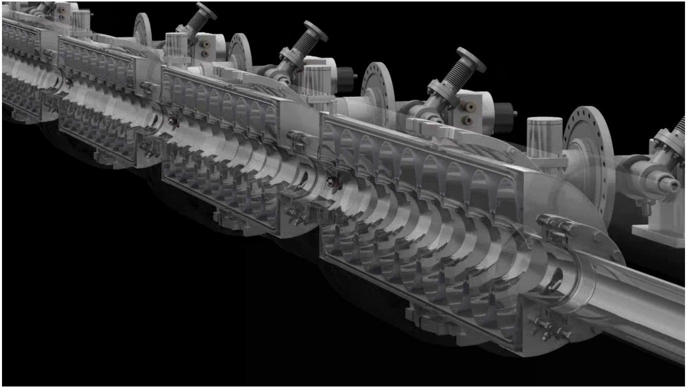
Linear Particle Accelerator (Linac)

New International Linear Collider (ILC) project (planning stage)



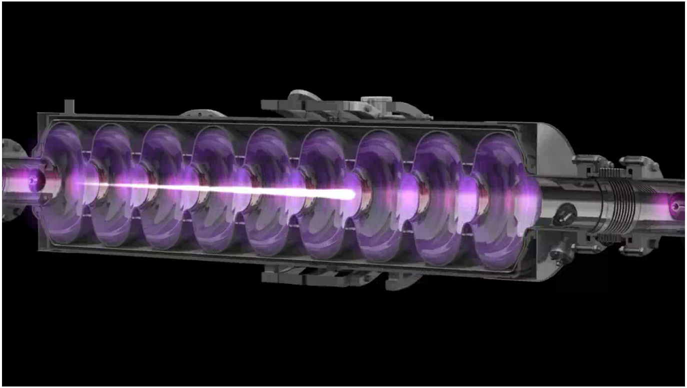
Structure design: two linacs throw particles toward each other at nearly the speed of light.

Linear Particle Accelerator (Linac)



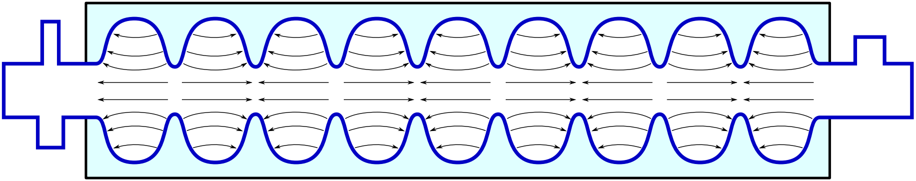
The accelerator is a sequence of several TESLA cavities.

Linear Particle Accelerator (Linac)



The TESLA cavities are composed of 9 niobium cells.

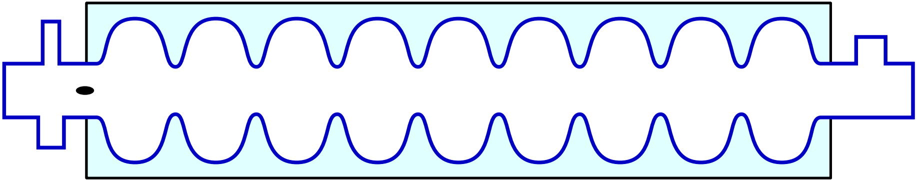
Particle acceleration in a TESLA Cavity



Electric field of the TM_{010} π-mode

- Like a vibrating string the cavity can operate at different frequencies and the field will have different shapes (modes)
- By selecting the frequency one selects the operating mode
- To achieve acceleration the fundamental Transverse Magnetic (TM) mode is used

Particle acceleration in a TESLA Cavity



Schematic of the acceleration of the beam

- Synchronization between particle bunches and field is fundamental
- Geometry is responsible for the operating frequency

⇒ Exact geometry representation is paramount

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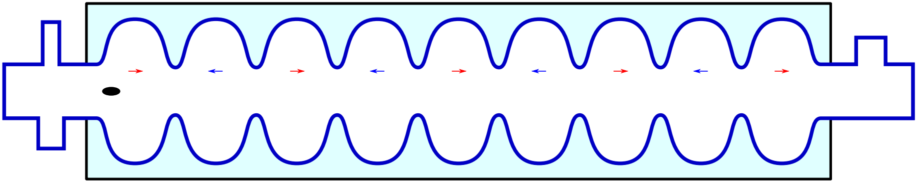

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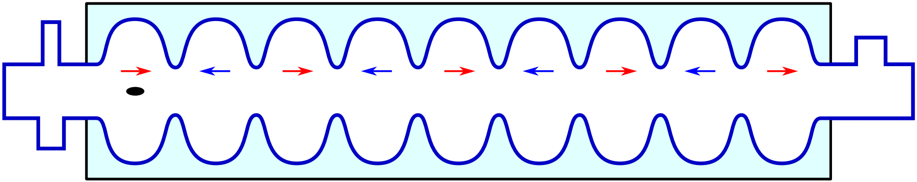

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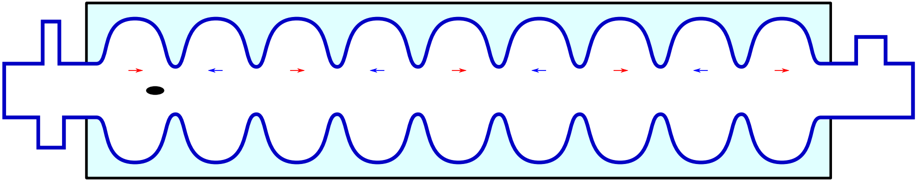

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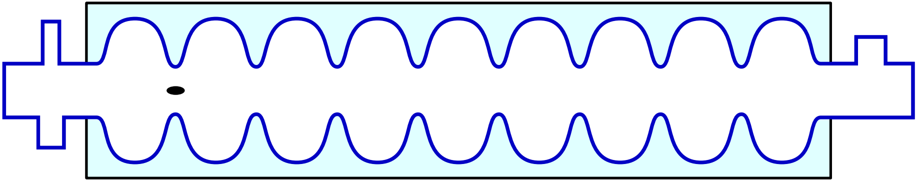

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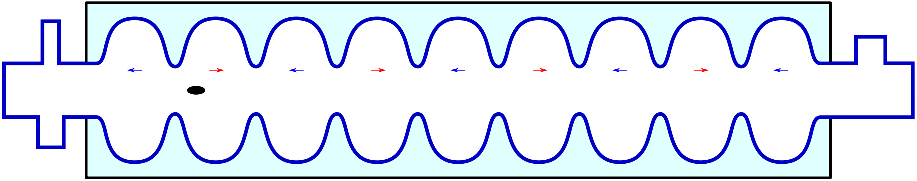

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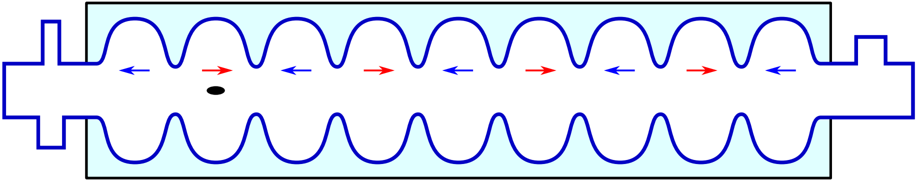

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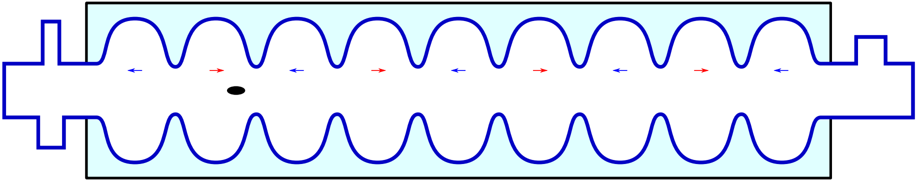

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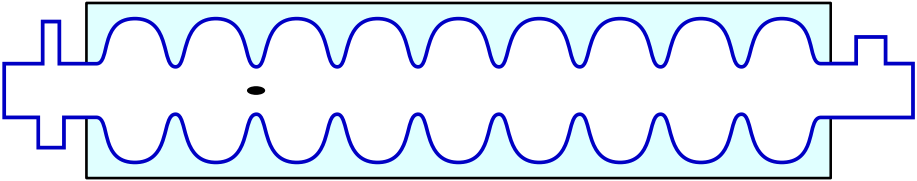

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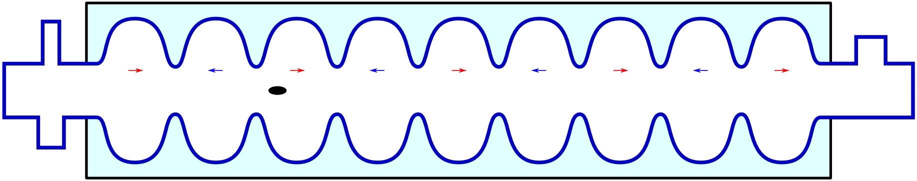

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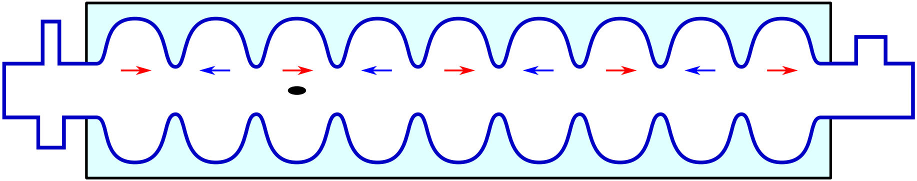

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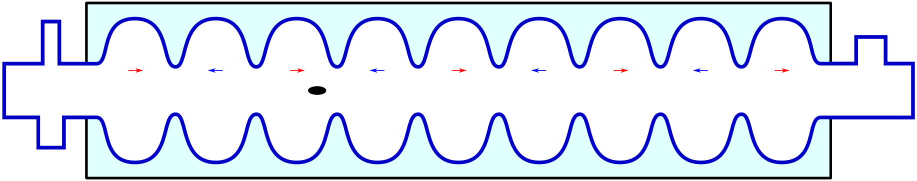

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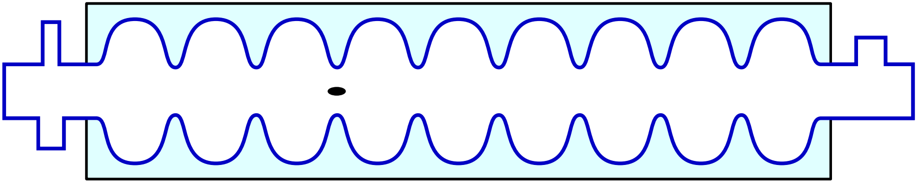

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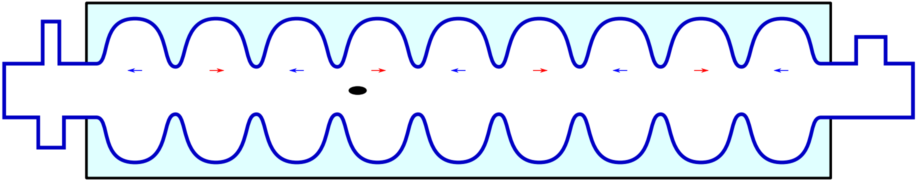

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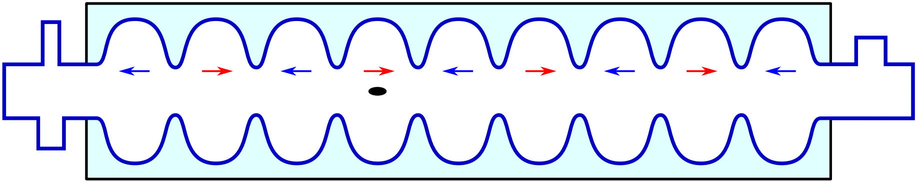

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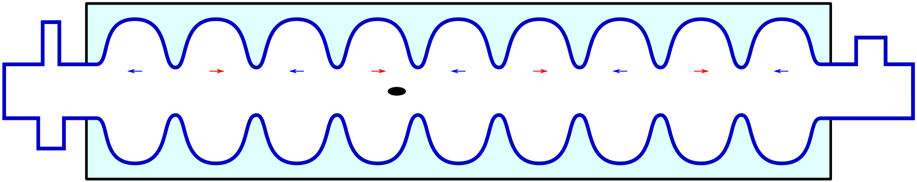

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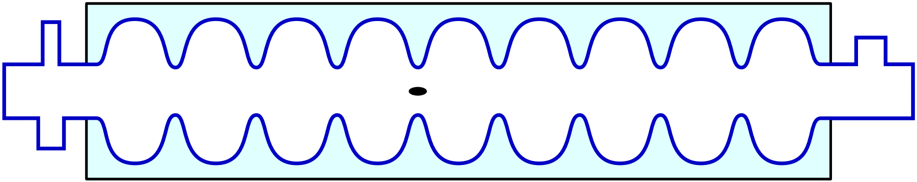

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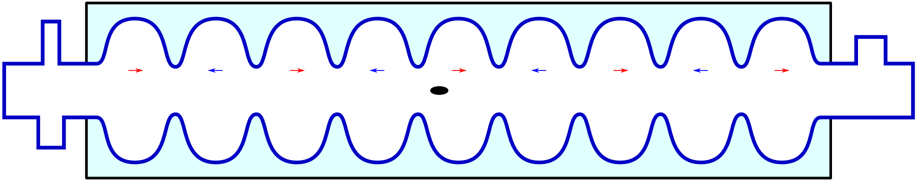

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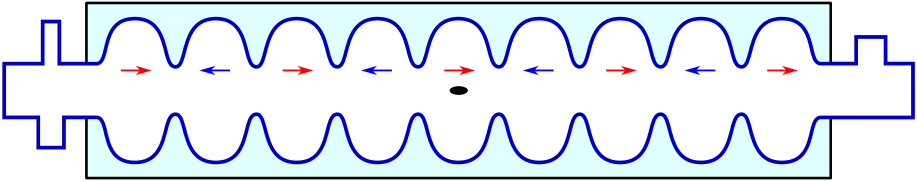

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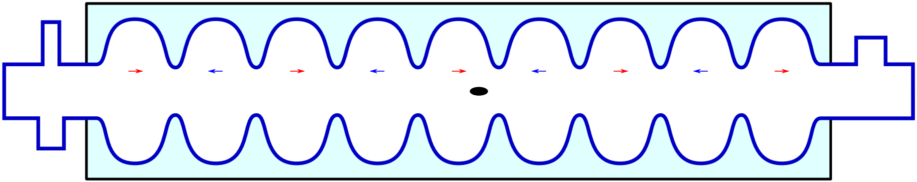

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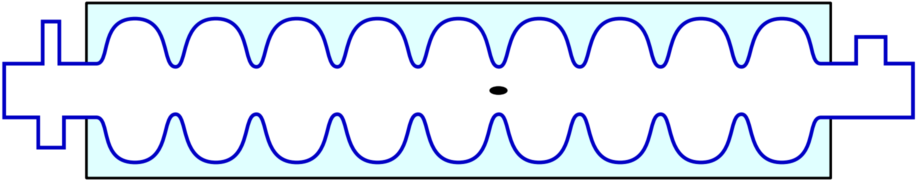

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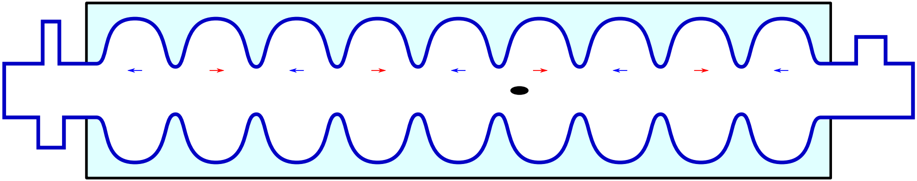

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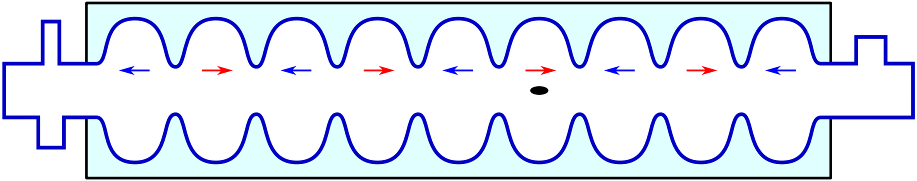

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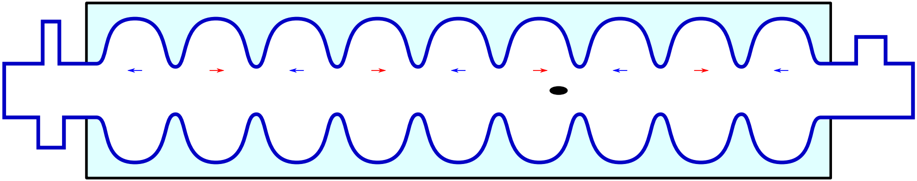

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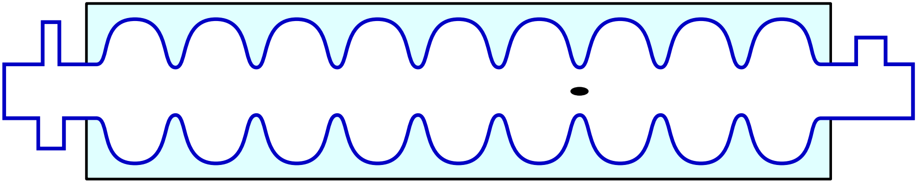

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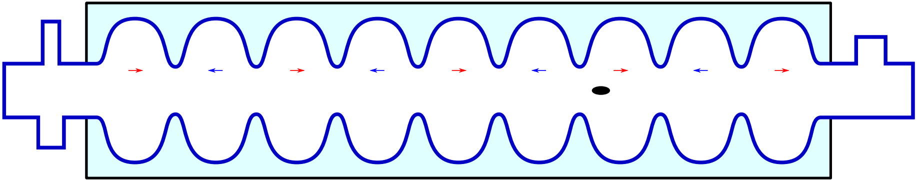

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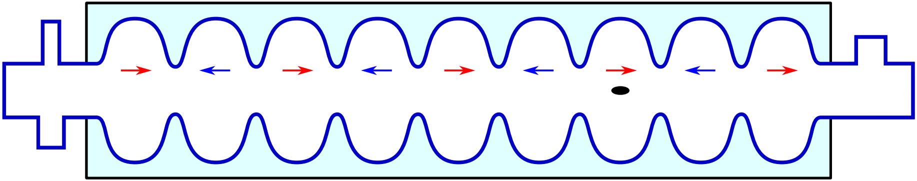

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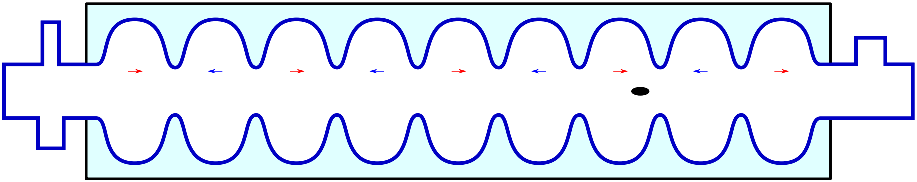

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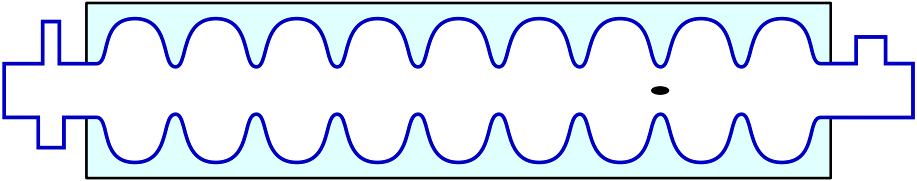

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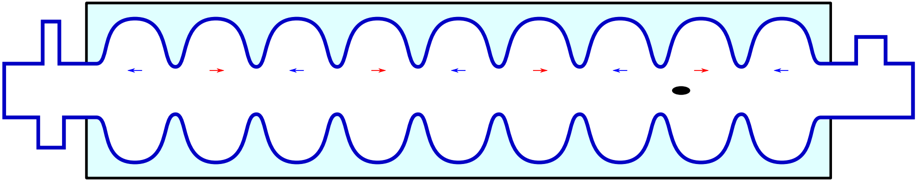

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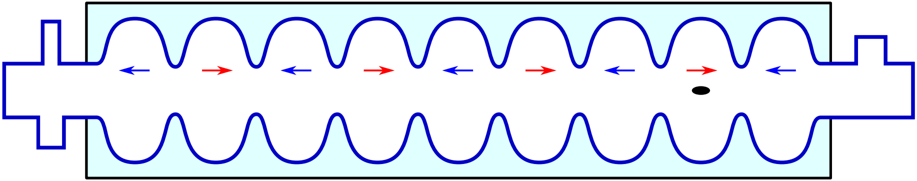

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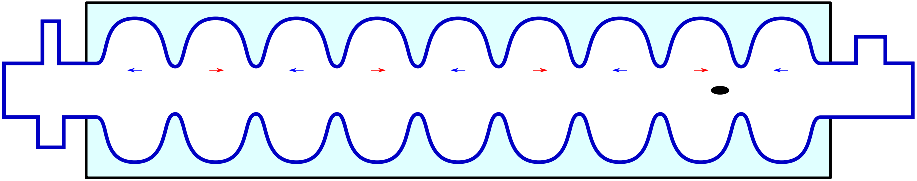

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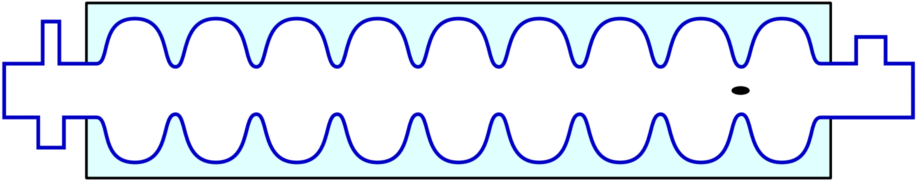

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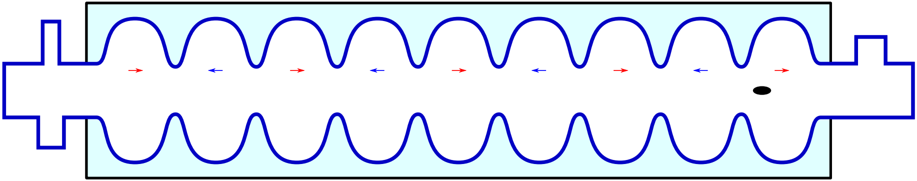

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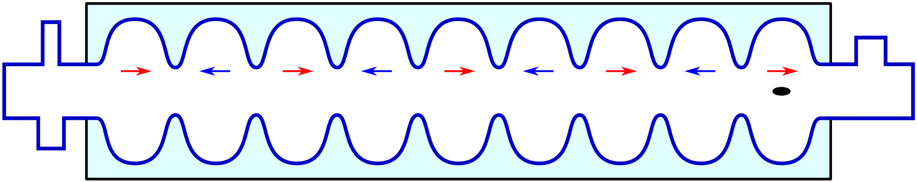

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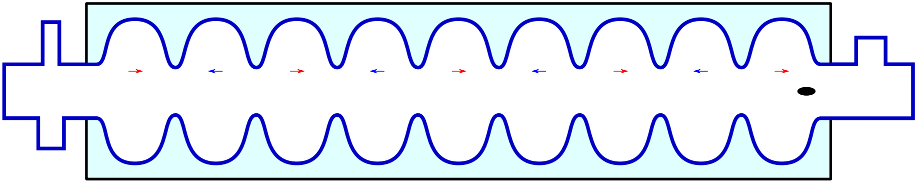

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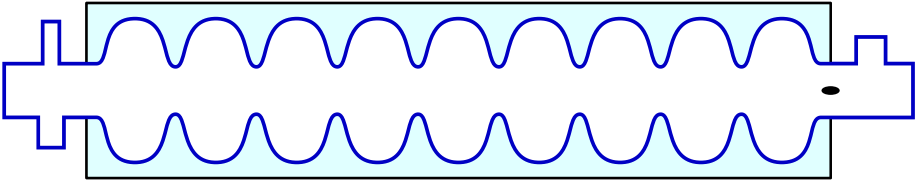

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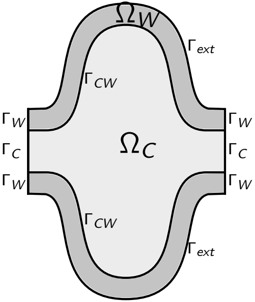
Multiphysics model

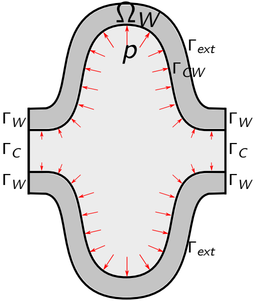
Relevant phenomenon: Lorentz detuning

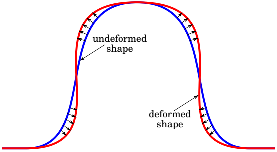
The electromagnetic field creates a pressure on the cavity wall

$\Rightarrow$ They are slightly ($\sim$10nm) deformed $\Rightarrow$ Shift of the frequency

- Electromagnetic problem in Ω_C
- Linear elasticity problem in Ω_W







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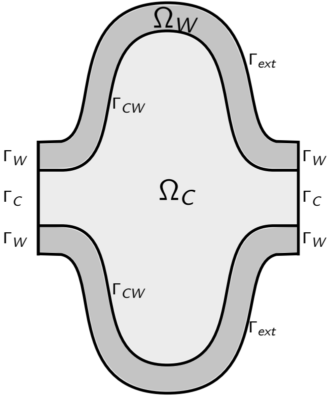
Electromagnetic problem



Maxwell's eigenvalue problem

$$\begin{cases} \nabla \times \left(\dfrac{1}{\mu_0} \nabla \times \mathbf{E} \right) = \omega^2 \varepsilon_0 \mathbf{E} & \text{in } \Omega_C \\ \mathbf{E} \times \mathbf{n}_c = 0 & \text{on } \Gamma_{CW} \\ \left(\dfrac{1}{\mu_0} \nabla \times \mathbf{E} \right) \times \mathbf{n}_c = 0 & \text{on } \Gamma_C \end{cases}$$

- electric field $\mathbf{E}$ → eigenvector, angular frequency ω → eigenvalue
- *metallic* boundary conditions on Γ_{CW} → zero tangential component
- *homogeneous Neumann* boundary conditions on Γ_C → due to the symmetry

ε_0 electric permittivity, μ_0 magnetic permeability of vacuum

Our application: mode $\mathbf{E}_0$ with the *smallest frequency* ω_0

The second order time-harmonic Maxwell's equation

The equation can be derived from the *classical system of Maxwell's equations*:

$$\begin{aligned} \varepsilon \frac{\partial \mathbf{E}}{\partial t} - \nabla \times \mathbf{H} + \sigma \mathbf{E} &= 0 \quad \text{(Ampère-Maxwell theorem)} \\ \mu \frac{\partial \mathbf{H}}{\partial t} + \nabla \times \mathbf{E} &= 0 \quad \text{(Faraday's law)} \\ \nabla \cdot (\varepsilon \mathbf{E}) &= \rho \quad \text{(Gauss' theorem)} \\ \nabla \cdot (\mu \mathbf{H}) &= 0 \quad \text{(Gauss' theorem for magnetism)} \end{aligned}$$

- assuming that the initial conditions already satisfy the two Gauss' theorems $\Rightarrow$ they are automatically satisfied for all time,
- using the first two equations we obtain

$$\nabla \times \left(\frac{1}{\mu} \nabla \times \mathbf{E} \right) + \varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} + \sigma \frac{\partial \mathbf{E}}{\partial t} = 0,$$

- by assuming $\mathbf{E}(\mathbf{x}, t) = \mathrm{Re}(\tilde{\mathbf{E}}(\mathbf{x}) e^{i\omega t})$, $\sigma = 0$, $\varepsilon = \varepsilon_0$, $\mu = \mu_0$, we get

$$\nabla \times \left(\frac{1}{\mu_0} \nabla \times \tilde{\mathbf{E}} \right) - \omega^2 \varepsilon_0 \tilde{\mathbf{E}} = 0.$$

- Dropping the $\tilde{}$ symbol

$$\nabla \times \left(\frac{1}{\mu_0} \nabla \times \mathbf{E} \right) - \omega^2 \varepsilon_0 \mathbf{E} = 0$$

- we multiply each term by a (vector) *test function*

$$\mathbf{v} \in V = \{\mathbf{v} \in H(\text{curl}, \Omega_C), \mathbf{v} \times \mathbf{n} = 0 \text{ on } \Gamma_{CW}\}$$

($H(\text{curl}, \Omega_C)$ is the space of square integrable functions whose curl is also square integrable)

- we integrate using for the second term the *integration by parts formula*

$$\int_{\Omega_C} (\nabla \times \mathbf{u}) \cdot \mathbf{v} \, dx = \int_{\Omega_C} (\nabla \times \mathbf{v}) \cdot \mathbf{u} \, dx - \int_{\Gamma} (\mathbf{u} \times \mathbf{n}) \cdot \mathbf{v} \, d\sigma.$$

We obtain

$$\int_{\Omega_C} \left[\left(\frac{1}{\mu} \nabla \times \mathbf{E} \right) \cdot (\nabla \times \mathbf{v}) - \omega^2 \varepsilon_0 \mathbf{E} \cdot \mathbf{v} \right] dx - \int_{\Gamma} \left(\frac{1}{\mu} \nabla \times \mathbf{E} \times \mathbf{n} \right) \cdot \mathbf{v} \, d\sigma = 0.$$

- Dropping the $\sim$ symbol

$$\nabla \times \left(\frac{1}{\mu_0} \nabla \times \mathbf{E} \right) - \omega^2 \varepsilon_0 \mathbf{E} = 0$$

- we multiply each term by a (vector) *test function*

$$\mathbf{v} \in V = \{ \mathbf{v} \in H(\text{curl}, \Omega_C), \mathbf{v} \times \mathbf{n} = 0 \text{ on } \Gamma_{CW} \}$$

($H(\text{curl}, \Omega_C)$ is the space of square integrable functions whose curl is also square integrable)

- we integrate using for the second term the *integration by parts formula*

$$\int_{\Omega_C} (\nabla \times \mathbf{u}) \cdot \mathbf{v} \, dx = \int_{\Omega_C} (\nabla \times \mathbf{v}) \cdot \mathbf{u} \, dx - \int_{\Gamma} (\mathbf{u} \times \mathbf{n}) \cdot \mathbf{v} \, d\sigma.$$

We obtain

$$\int_{\Omega_C} \left[\left(\frac{1}{\mu_0} \nabla \times \mathbf{E} \right) \cdot (\nabla \times \mathbf{v}) - \omega^2 \varepsilon_0 \mathbf{E} \cdot \mathbf{v} \right] dx = 0.$$

Electromagnetic problem → elasticity problem

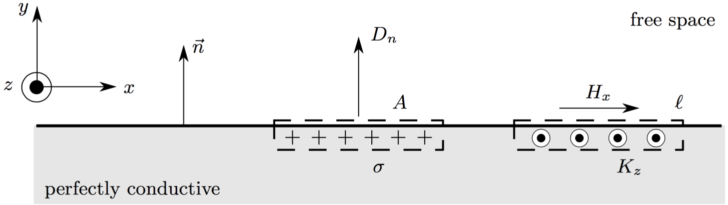
Radiation pressure on Γ_{CW} produced by the electromagnetic field:

$$p = -\frac{1}{4}\varepsilon_0 \left(\mathbf{E}_0 \cdot \mathbf{n}_c\right) \cdot \left(\mathbf{E}_0^* \cdot \mathbf{n}_c\right) + \frac{1}{4}\mu_0 \left(\mathbf{H}_0 \times \mathbf{n}_c\right) \cdot \left(\mathbf{H}_0^* \times \mathbf{n}_c\right)$$

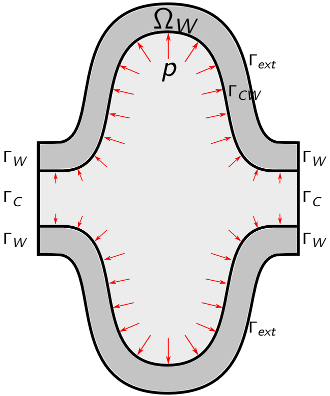
It depends on

- the normal component of the electric field,
- the tangential component of the magnetic field.

It is a time-constant value.



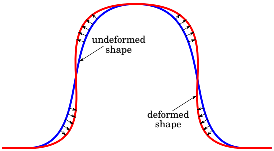
Linear elasticity problem

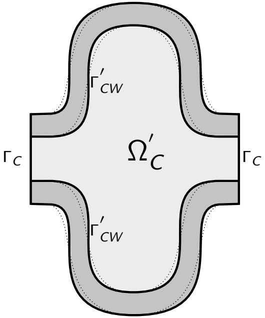


$$
\begin{cases}
\nabla \cdot \left(2\eta\nabla^{(S)}\mathbf{u} + \lambda \mathbf{I} \nabla \cdot \mathbf{u}\right) = 0 & \text{in } \Omega_W \\
\mathbf{u} = 0 & \text{on } \Gamma_W \\
\left(2\eta\nabla^{(S)}\mathbf{u} + \lambda \mathbf{I} \nabla \cdot \mathbf{u}\right) \cdot \mathbf{n}_w = p \cdot \mathbf{n}_w & \text{on } \Gamma_{CW} \\
\left(2\eta\nabla^{(S)}\mathbf{u} + \lambda \mathbf{I} \nabla \cdot \mathbf{u}\right) \cdot \mathbf{n}_w = 0 & \text{on } \Gamma_{ext}
\end{cases}
$$

- $\mathbf{u} \rightarrow$ displacement of each point
- $\eta, \lambda \rightarrow$ Lamé parameters of the material
- $\nabla^{(S)}\mathbf{u} = \frac{1}{2}(\nabla\mathbf{u} + \nabla\mathbf{u}^T)$ symmetric gradient

Deformed cavity





- Compute the deformation

$$\Omega'_W = \{\mathbf{x} + \mathbf{u}(\mathbf{x}), \mathbf{x} \in \Gamma_W\}$$

$$\Gamma'_{CW} = \{\mathbf{x} + \mathbf{u}|_{\Gamma_{CW}}(\mathbf{x}), \mathbf{x} \in \Gamma_{CW}\}$$

- Solve again the Maxwell's eigenvalue problem but in the *deformed cell* Ω'_C
- Compute the shift of the frequency (Lorentz detuning) $\Delta\omega_0 = |\omega_0 - \omega'_0|$

Outline

Strategy

- Geometry definition: Choose a representation of the domain
- Galerkin approach: Choose a discrete representation of the functional spaces where we look for our unknowns (standard example: Finite elements)

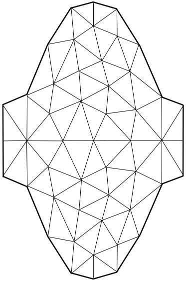
In particular:

- Take into account the physical properties of the fields (example: tangential continuity of $\mathbf{E}$)
- $\Gamma'_{CW} = \{\mathbf{x} + \mathbf{u}|_{\Gamma_{CW}}(\mathbf{x}),\, \mathbf{x} \in \Gamma_{CW}\}$ elasticity problem: structural problem, the unknown $\mathbf{u}$ is the displacement of the geometry

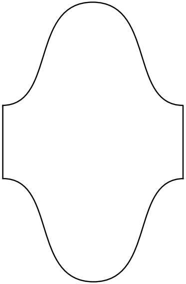
Isogeometric analysis (IGA)

The deformed domain is obtained by adding the displacement to the original domain $\Omega'_W = \Omega_W + \mathbf{u}$

- Approximated geometry + Finite elements
- CAD exact geometry + Finite elements + interpolation
- CAD exact geometry + NURBS (IGA)



FEM geometry



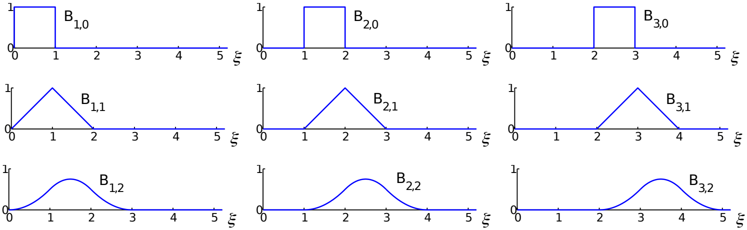
CAD geometry

Idea: use the CAD basis functions (B-Splines, NURBS) not only for the geometry but also for the discretization space!

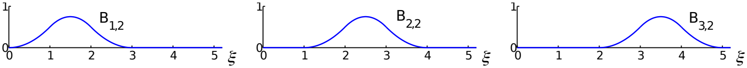
Given a *non-uniform knot vector* $\{\xi_1, ..., \xi_{n+p+1}\}$, in the parametric domain, B-spline basis functions are:

$$B_{i,0}(\xi) = \begin{cases} 1 \text{ if } \xi_i \leq \xi < \xi_{i+1} \\ 0 \text{ otherwise.} \end{cases}$$

$$B_{i,p}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} B_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} B_{i+1,p-1}(\xi).$$

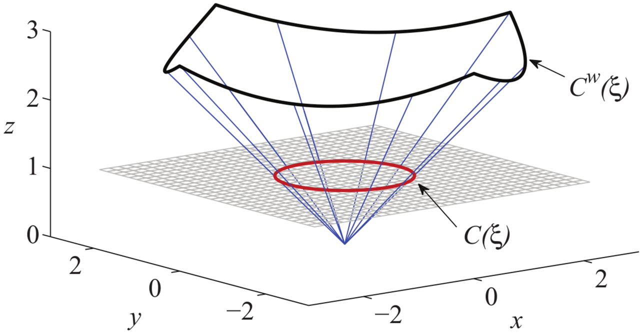


$$B_{i,p}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} B_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} B_{i+1,p-1}(\xi).$$



- The support of $B_{i,p}$ is (ξ_i, ξ_{i+p+1})
- The functions $B_{i,p}$ are non-negative and enjoy the partition–of–unity property
- The derivative of a B-spline is still a B-spline

A NURBS curve in $\mathbb{R}^2$ is the projection of a B-spline in $\mathbb{R}^3$



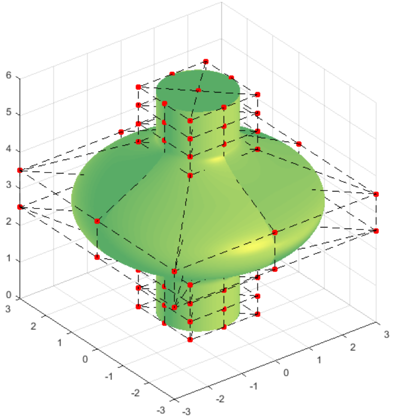
$$\color{red}{C(\xi)} = \sum_{i=1}^{n} \mathbf{C}_i \frac{w_i B_{i,p}(\xi)}{\sum_{\hat{i}=1}^{n} w_{\hat{i}} B_{\hat{i},p}(\xi)} = \sum_{i=1}^{n} \mathbf{C}_i N_{i,p}(\xi)$$

Isogeometric analysis (IGA)

TESLA geometry definition - computed via the NURBS package

To construct the TESLA cavity geometry: NURBS toolbox.

- It is a collection of routines for Octave and Matlab for the creation and manipulation of NURBS, developed by M. Spink in 2000.
- From 2009, extended and maintained by Carlo de Falco and Rafael Vázquez.

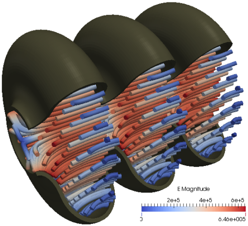


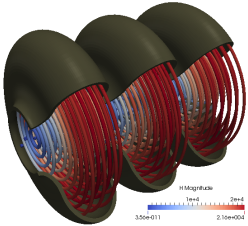
Isogeometric analysis (IGA)

The Electric field - computed via the GeoPDEs package

To solve the equations: GeoPDEs

- Free package for Octave and Matlab for solving PDEs using the IGA.
- Originally (2009) developed in Pavia, by C. de Falco, A. Reali, and R. Vázquez.
- We employed this package to reproduce some of the results obtained by J. Corno.

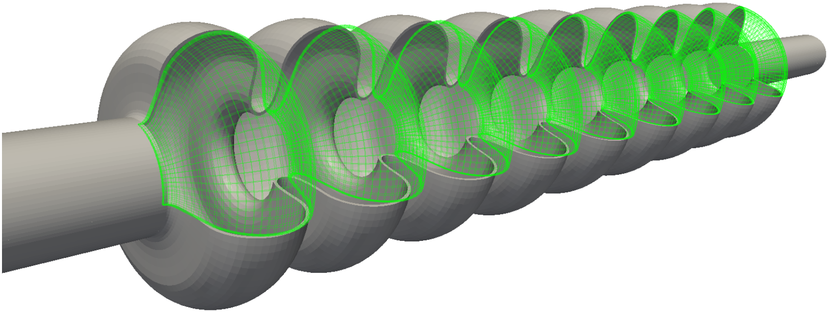




The field lines for the electric (left) and magnetic (right) field.
The solutions have been rescaled so that the total energy is 1J.

Isogeometric analysis (IGA)

The deformation



Deformation of the cavity walls (rescaling factor $2 \cdot 10^5$).

Outline

Summary

- structure and physics of a Linear Collider,
- TESLA cavity and the multiphysics problem,
- mathematical formulation through a coupled electromagnetic and elastic problem
  - Maxwell's equations $\rightarrow$ double curl time-harmonic PDE for $(\mathbf{E}_0, \omega_0)$
  - elasticity deformation due to radiation pressure $\rightarrow$ displacement $\mathbf{u}$
- Lorentz detuning
  - initial cavity $\rightarrow$ corresponding ω_0
  - radiation pressure $\rightarrow$ elastic deformation of the walls
  - new deformed cavity $\rightarrow$ new ω_0'
  - detuning effect $|\omega_0 - \omega_0'|$
- isogeometric analysis: same framework used for both geometry and numerical analysis,
- GeoPDEs and NURBS Octave packages allow an easy implementation.

Outlook

First step:

- solve the eigenvalue problem in the initial cavity $\longrightarrow (\mathbf{E}_0, \omega_0) \longrightarrow$ extract $\mathbf{E}_b = \mathbf{E}|_{\Gamma_C} \rightarrow$ use that as BC to excite the mode ω_0

Second step:

1. set $\Omega_C^0 = \Omega_C$
2. find $\mathbf{E} \in V^0 = \{\mathbf{v} \in H(\text{curl}, \Omega_C^0), \mathbf{v} \times \mathbf{n} = 0 \text{ on } \Gamma_{CW}\}$ s.t.

$$\int_{\Omega_C^0} \left(\frac{1}{\mu_0} \nabla \times (\mathbf{E} + \mathbf{E}_b^0) \right) \cdot (\nabla \times \mathbf{v})\, dx - \omega_0^2 \varepsilon_0 \int_{\Omega_C^0} (\mathbf{E} + \mathbf{E}_b^0) \cdot \mathbf{v}\, dx = 0$$

   for all $\mathbf{v} \in V^0$, with $\mathbf{E}_b^0$ an extension of the boundary condition over Ω_C^0
3. determine the deformed cavity Ω_C^1 (note that Γ_C won't move because $\mathbf{u} = 0$ on Γ_C anyway)
4. repeat from 2 with the new cavity using the same ω_0

Outlook

- sequence of domains Ω_C^0, Ω_C^1, Ω_C^2, Ω_C^3 ...
- natural question: does this sequence converge?
- if yes, what about Ω_{inf}? What about the resulting field $\mathbf{E}_{\text{inf}}$? is this convergence fast?

Thank you for your attention!

Outlook

- sequence of domains Ω_C^0, Ω_C^1, Ω_C^2, Ω_C^3 ...
- natural question: does this sequence converge?
- if yes, what about Ω_{inf}? What about the resulting field $\mathbf{E}_{\text{inf}}$? is this convergence fast?

Thank you for your attention!

References

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R. Vázquez. *A new design for the implementation of isogeometric analysis in Octave and Matlab: GeoPDEs 3.0*. Comput. Math. Appl., 72(3):523-554, 2016.

J. Corno, C. de Falco, H. De Gersem, S. Schöps. *Isogeometric simulation of Lorentz detuning in superconducting accelerator cavities*. Computer Physics Communications, 201, 1-7, 2016.